A Portfolio's Risk - Return Analysis

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I. INTRODUCTION

The design of this paper is to illustrate a portfolio's performance via a series of standard risk measurements, while at the same time answering the question of whether a portfolio achieves sufficient return in exchange for the risk taken. The benchmark against which the portfolio is being measured is the S&P 500 index. The S&P 500 was chosen due to its universal acceptability.

It is acknowledged that some of the risk measures require comparison with otherportfolios or portfolios.

A summary of the results can be found in Appendix A, while the data utilized can be found in Appendix C. Appendix B is a graph of the regression of the returns as the dependent variable on the S&P 500 index returns, or what are being called here the market returns, as the independent variable.

Risk can be calculated in two fundamentally different ways, *ex post* and *ex ante*. Ex post is an analysis of risk using historical data. It answers the question of how risky the portfolio or portfolio under consideration has been over a specific period of time. Ex ante risk is an estimate or forecast of future risk using forecasted data. In this report we will be concerned exclusively with the ex post risk behavior.

A number of statistical results, such as covariance, and risk indicators, such as the Sharpe ratio, are influenced by the magnitude of the data utilized and time periods selected. It is stressed that the data is annual data for the period previously stated, 1981 through 2008, with the exception of the drawdown calculations where the utilization of monthly data is more appropriate.

A number of the calculations utilize a risk free rate of return. For the purpose of this paper the risk free rate of return will be assumed to be an annual rate of 5 percent. Although a lower rate is currently available in today's financial markets, the selection is both conservative and practical given the assumption that interest rates from a historical perspective are currently abnormally low.

Where a minimum required rate of return is required, a rate of return that is greater than zero is considered to be the acceptable minimum.

Many parameters of risk assume that the data employed has a normal distribution and that kurtosis and skewness, if present, can be problematic. Furthermore, as has been pointed out by Benoit Mandelbrot and Nassim Nicholas Taleb in several articles, normal curves do not take into account large price shifts. Although the portfolio's returns are slightly skewed and subject to some kurtosis, as is demonstrated in the analysis, the lack of normality is unlikely to be detrimental to the analysis.

II. BENCHMARK STATISTICS

Capture Indicators

Up and down capture indicators are a measure of how well the portfolio is able to replicate or improve on positive benchmark returns, while at the same time indicating the extent to which the portfolio's returns were adversely affected by negative benchmark returns.

To calculate the up capture, you ignore all time periods where the benchmark S&P 500 return is zero or negative. The up capture is then the quotient of the portfolio's average return divided by the average return of the resulting S&P 500 series for only those periods where the S&P 500 index has produced a positive return.

In other words, the up capture indicator divides the portfolio's average return for those periods where the S&P 500 index has a positive return by the average return of the S&P 500 over the same period , i.e., where the S&P 500 index returns are positive. The down capture is calculated analogously, dividing the average annual return of the portfolio by the average annual S&P 500 index return utilizing only the data from those periods where the annual returns of the benchmark S&P 500 index are negative.

Up Capture Indicator

$$\text{Up capture indicator} = \frac{\overline{r_p^+}}{\overline{r_m^+}}$$

$\overline{r_p^+}$ is the geometric average of the portfolio's return over the periods in which the S&P 500 index return is positive.

$\overline{r_m^+}$ is the geometric average of the S&P 500's return over the periods in which the S&P 500 index return is positive.

Down Capture Indicator

$$\text{Down capture indicator} = \frac{\overline{r_p^-}}{\overline{r_m^-}}$$

$\overline{r_p^-}$ is the geometric average of the portfolio's return over the periods in which the S&P 500 index return is negative.

$\overline{r_m^-}$ is the geometric average of the S&P 500's return over the periods in which the S&P 500 index return is positive.

Up Number ratio

The up number ratio measures the percentage of positive returns out of total number of returns for the portfolio during those times when the returns for the benchmark S&P 500 index are also positive. The closer this number is to 100 percent the better.

Down Number ratio

The down number ratio measures the percentage of negative returns out of total number of returns for the portfolio during those times when the returns for the benchmark S&P 500 index are also negative. The closer this number is to 0 percent the better; although for high correlation this number could approach 100 percent. Note that there is a high correlation between the portfolio and the S&P 500 index.

The Down Number for the portfolio is: 33.3 percent.

Therefore, if the S&P index had a negative return for the year, then 33 percent of the time the portfolio is also showed a negative return.

Up Percentage ratio

The up percentage ratio measures the percentage of periods in which the excess return of the portfolio over the S&P 500 index is positive when the S&P 500 index is positive. In other words, how often does the portfolio outperform in a rising market?

The Up Percentage ratio for the portfolio is: 59 percent

Therefore, the portfolio outperforms the S&P index 59 percent of the time, even when the index has a positive gain.

Down Percentage ratio

The down percentage ratio measures the percentage of periods, in which the excess return of the portfolio over the S&P 500 index is positive when the S&P 500 index is negative. In other words, how often does the portfolio outperform the S&P 500 index during those times when the S&P500 posts a negative return?

The Down Percentage ratio for the portfolio is: 100 percent.

Therefore for 100 percent of the time when the S&P 500 index posts a negative return, the portfolio has outperformed the index with a positive excess return. However, a word of caution is in order here because the number of data points for when the S&P 500 index is negative for the year is relatively small. The specific data can be found in Appendix B.

Percentage Gain ratio

The percentage gain ratio is defined as the number of times the portfolio's return is greater than zero divided by the number of S&P 500 returns greater than zero.

The percentage gain for the portfolio is: 118 percent

In other words the portfolio has had a greater number of positive returns than the S&P 500 index.

Summary

As indicated by the various capture statistics, the portfolio exceeds the positive return offered by the S&P 500 index when the index is positive and yet manages to escape much of the loss incurred by the S&P 500. In other words, the positive gains by the portfolio are considerably higher than those of the index, while at the same time giving up considerably less ground when the index turns negative.

III. RISK RATIOS

Standard Deviation

Volatility or risk can either be measured by using the variance or standard deviation of a portfolio. As a general rule the higher the volatility the riskier the security. In other words, volatility refers to the amount of uncertainty or risk regarding the degree of change in the return of a portfolio. Higher volatility means that returns are generally spread out over a larger range of values. Furthermore, greater volatility can result in returns that change dramatically over a short time period. The changes can be either positive or negative in nature. A lower volatility means that a security's value does not fluctuate as dramatically and changes in value come in smaller increments.

For analysis purposes it is more convenient to use non-squared units of return; therefore the basic unit of risk is the standard deviation (σ) , defined as the square root of the variance(σ^2) , where the variance is the average squared deviation of returns from the mean.

Note that in calculating the standard deviation, n rather than n-1 is used in the denominator. Although it makes little difference in large samples, the CFA Institute has effectively reinforced the use of n rather than n-1.

The variance of the portfolio (${\sigma_p}^2$) is: 151

The variance of the S&P 500 market index (${\sigma_m}^2$) is: 321

The standard deviation of the portfolio (σ_p) is: 12.3

The standard deviation of the S&P 500 index (σ_m) is: 17.9

As is evident from the two standard deviation figures, the portfolio enjoys roughly 68 percent less volatility than the S&P 500.

The Sharpe Ratio

Although Dr. William Sharpe has on many occasions disdained its promotional use as a singular predictor of past performance, the Sharpe ratio has become a cornerstone of modern finance to measure how well the return of an asset compensates an investor for the risk undertaken.

The Sharpe ratio is defined as the return on the portfolio (r_p) or benchmark (r_m) less the risk free rate of return (r_f) divided by standard deviation of the portfolio (σ_p) or benchmark (σ_m).

$$\textbf{Sharpe ratio} = \frac{r_p - r_f}{\sigma_p} \qquad \textbf{Sharpe ratio S\&P 500} = \frac{r_m - r_f}{\sigma_m}$$

Therefore, for the purposes of this study, the mean return is either the arithmetic or geometric mean (both calculations are provided) less the risk free rate of return, which is assumed to be 5 percent, divided by the appropriate standard deviation.

Unfortunately, the Sharpe ratio is often carelessly used as a way to refer to the level of risk adjusted returns when in actuality it provides only the volatility of adjusted returns.

Other ratios such as the bias ratio have recently been introduced to handle cases where the observed volatility may be an especially poor proxy for the risk inherent in a time-series of observed returns.

The Sharpe Ratio for the portfolio using the arithmetic mean is: 0.96

The Sharpe Ratio for the S&P 500 index using the arithmetic mean is: 0.65

The Sharpe Ratio for the portfolio using a geometric mean is: 0.90

The Sharpe Ratio for the S&P 500 index using a geometric mean is: 0.56

Since the Sharpe Ratio is a risk adjusted measure of performance, the higher the Sharpe Ratio the better. As is obvious, the portfolio far outperforms the S&P 500 index in terms of risk adjusted return as measured by the Sharpe Ratio.

Risk-Adjusted Return: M^2

Although it is easy to rank performance with the Sharpe ratio, it is difficult to judge the size of relative performance. A risk-adjusted return measure that offers a better understanding of risk-adjusted performance is a statistic referred to as M^2.

It is called M^2 not because any element of the calculation is squared but because it was first proposed by the partnership of Leah Modigliani and her grandfather Franco Modigliani. The statistic is defined as the average return of the portfolio plus the Sharpe Ratio multiplied by the difference between the standard deviation of benchmark and the portfolio.

$$M^2 = r_p + SR \times (\sigma_m - \sigma_p)$$

The statistic represents the return on an investment when that investment is adjusted to the same volatility or risk level as the overall market with which the investment is being compared. M^2 statistics that exceed their benchmark market returns indicate that the investment returned more for the risk undertaken than the market did. The converse is that when the M^2 statistic is less than the benchmark market return, it indicates that the investment or portfolio did not return as much as the market.

The M^2 statistic for the portfolio is: 22.14 percent.

Compared to the average return of the S&P 500 of 11.61 percent, it is easy to see that the portfolio offer nearly twice the return for the same amount of risk as determined by M^2.

M^2 Excess Returns

Exactly the same argument can be applied to M^2 excess returns as to normal excess returns. Using an arithmetic return, the M^2 excess return is defined as follows:

$$M^2 \text{ excess return} = M^2 - r_m$$

The M^2 Excess Return for the portfolio is: 10.3 percent.

IV. REGRESSION ANALYSIS

Basic Regression

To gain additional information about the performance of the portfolio it is necessary to regress the returns of the portfolio r_p against those of the benchmark S&P 500 index (r_m). A scatter plot of the points is shown in Appendix B.

The regression equation is defined as:

$$r_p = a + b \times r_m + e$$

The intercept of the regression equation with the vertical axis is noted by a. Although the "a" is often referred to as the alpha of the regression, It should not be confused with the Jensen Alpha to be discussed later.

The b is the slope or gradient of the regression equation.

The e is an error term measuring the vertical distance between the return predicted by the equation and the actual return. The residuals for the regression are plotted in Appendix B.

The regression for the portfolio where the independent variable is the return on the S&P 500 benchmark and the dependent variable is the portfolio's return is calculated to be:

$$Y = 10.006 + 0.5821\ X$$

Looking at the equation, it is easy to see that the intercept is 10.006 and slope of the equation, which is also the beta, is 0.5821.

Capital Asset Pricing model (CAPM)

Using the CAPM model, we can factor in the risk free rate of return and calculate a new alpha and beta, only this time the alpha would be what is referred to as Jensen's Alpha. A CAPM beta would require monthly risk free rates of return. However, if the risk free rate is constant, as it is in this case at 5 percent, then the regression beta and the CAPM beta are identical.

The CAPM regression is defined as:

$$r_p - r_f = \alpha + \beta \times (r_m - r_f) + \varepsilon$$

Where: r_p = portfolio's return

r_f = the risk free rate of return (assumed here to be 5 percent)

r_m = the return on the S&P 500 index

β = the systematic or non-diversifiable risk

α = vertical axis intercept also known as Jensen's alpha

ε = the error term

The beta using the return on the portfolio is 0.5821

Jensen's Alpha

Michael Jensen's alpha is the intercept of the regression equation in CAPM and is in effect the excess return adjusted for beta or systematic risk. Systematic risk is also that risk which is referred to as market risk or that risk which cannot be diversified away.

Portfolio managers often talk in terms of alpha to describe their added value. Yet, rarely are they referring to either the regression or even Jensen's alpha. In all likelihood they are referring to their excess return above the benchmark. Confusingly, academics also frequently refer to excess return as the return above the risk free rate.

Ignoring the error term since these are ex post calculations, and using the fact that because the risk free rate is constant then the regression beta is the same as the CAPM beta, Jensen's alpha can be calculated as:

$$\alpha = r_p - [\, r_f + \beta \times (r_m - r_f)]$$

The Jensen alpha for the portfolio is: 15.61 percent.

What that means is that the return of the portfolio after adjusting for systematic risk, also referred as the beta, is 15.61 percent.

Bull and Bear Beta (β^+ and β^-)

It is not necessary to restrict regression lines to a best fit of all market returns, positive and negative. It is also possible to calculate a regression equation for only positive market returns thereby gaining information on the behavior of the portfolio in positive or "bull" markets. The same procedure using negative market returns yields the bear beta.

As has been mentioned previously, the number of data points for negative market returns is severely limited. Therefore the results should not be taken too seriously.

β^+ for the portfolio is equal to: 0.607

β^- for the portfolio is equal to: 0.823

Beta Timing Ratio

Ideally, a portfolio manager has a beta greater than 1 in a rising market and less than 1 in a falling market. To that end it would be indicative of predictive behavior with regard to timing asset allocation decisions. One ratio that portends this ability is the beta timing ratio, defined as:

$$\text{Beta Timing Ratio} = \frac{\beta+}{\beta-}$$

The monthly beta timing ratio for the portfolio is: 0.74.

The key point here is that whether you are measuring the portfolio's returns against positive or negative benchmark returns, the portfolio's risk is less than that of the overall market as measured by the S&P 500 index.

Covariance

Covariance measures the tendency of the portfolio and benchmark returns to move together. Therefore, a positive covariance indicates the returns are associated, that they move together. A negative covariance indicates the returns move in opposite directions. A low or near zero covariance would indicate no relationship between the portfolio and benchmark.

$$\text{Covariance} = \frac{\sum_{i=1}^{n}(r_p - \overline{r_p})\ x\ (r_m - \overline{r_m})}{n}$$

$\overline{r_p}$ = mean return of the portfolio = 16.77 percent

$\overline{r_m}$ = mean return of the S&P 500 index = 11.61 percent

The covariance of the portfolio with the S&P 500 is: 180.34

Because the covariance is large it can be concluded that the portfolio does follow in the direction of the S&P 500 index. However, covariance is also difficult to use for comparison purposes as described here because it is impacted by the absolute size of the returns.

Correlation

In isolation covariance is a difficult statistic to interpret. However, the covariance can be standardized as a value between 1 and -1 by dividing the covariance by the product of the portfolio's standard deviation and the standard deviation of the S&P 500.

$$\text{Correlation } \rho_{p,m} = \frac{Covariance}{\sigma_p \, x \, \sigma_m}$$

Note that correlation is also:

$$\rho_{p,m} = \frac{Systematic\ risk}{Total\ risk}$$

$$\rho_{p,m} = \frac{\beta \, x \, \sigma_m}{\sigma_p}$$

Therefore beta and correlation are linked by the formula:

$$\beta = \rho_{p,m} \text{ x } \frac{\sigma_p}{\sigma_m}$$

In other words, correlation measures the variability in the portfolio that is systematic compared to the total variability.

The correlation of the portfolio compared to the benchmark S&P 500 index is: 0.85

This confirms the high correlation between the portfolio's returns and those of the S&P 500 index.

Coefficient of Determination or R^2

The $\boldsymbol{R^2}$ of any regression analysis is the proportion of the variance in the portfolio that is related to the variance of the S&P 500 index. It is a measure of diversification. The closer R^2 is to 1 the more portfolio's variance is explained by the variance of the S&P 500. A low R^2 would indicate that returns are more scattered and would indicate a less reliable line of best fit leading to unstable alphas and betas.

R^2 for the regression of the data from the portfolio against the S&P 500 is: 0.72

Systematic Risk

Michael Jensen described beta as systematic risk. By multiplying beta by market risk, you obtain a measure of systematic risk calculated in the same units as variability, which is considered by some to be a better measure of systematic risk.

$$\text{Systematic risk } \sigma_s = \beta \, x \, \sigma_m$$

The systematic risk for the portfolio is: 10.43

Specific or residual risk

Residual or specific risk is not attributed to general market movements but is unique to the particular portfolio under consideration. It is represented by the standard deviation of the error term in the regression equation σ_e.

For the portfolio σ_e is: 6.51

Since specific risk and systematic risk are by definition independent, total risk can be defined as follows:

$$\text{Total risk}^2 = \text{systematic risk}^2 + \text{specific risk}^2$$

For the portfolio, total risk is: 12.3

Treynor Ratio

The Treynor ratio is similar to the Sharpe ratio except that the denominator is the systematic risk defined as the beta of the portfolio or portfolio under consideration. The Treynor ratio is not used as often as it should be because it ignores specific risk.

In the case of full diversification with no specific risk the Treynor and the Sharpe ratios will give the same ranking. Dr. Sharpe actually favored the Treynor ratio because he felt any value gained from being not fully diversified was transitory.

$$\text{Treynor ratio} = \frac{r_p - r_f}{\beta_p}$$

The Treynor ratio for the portfolio is: 8.18.

This equates to an 8.18 percent return per unit of systematic risk

Modified Treynor

A logical alternative, referred to as the Modified Treynor ratio, uses systematic risk σ_s in the denominator because it is more consistent with the Sharpe ratio.

$$\text{Modified Treynor} = \frac{r_p - r_f}{\sigma_s}$$

The modified Treynor ratio for the portfolio is: 1.13.

This equates to a 1.13 percent return per unit of systematic risk.

Appraisal ratio (Treynor-Black)

The appraisal ratio, first suggested by Treynor and Black (1973) is similar in concept to the Sharpe ratio, however it uses Jensen's alpha, which is excess return adjusted for systematic risk, in the numerator and specific risk, not total risk in the denominator. Keep in mind that once again that σ_e is the standard deviation of the error term in the regression equation.

$$\textbf{Appraisal Ratio} = \frac{\alpha}{\sigma_e}$$

The appraisal ratio for the portfolio is: 2.40

Modified Jensen

The modified Jensen ratio is Jensen's alpha is divided by systematic risk rather than specific risk. As a result, you get the systematic risk-adjusted return per unit of systematic risk.

$$\textbf{Modified Jensen} = \frac{\alpha}{\beta}$$

The modified Jensen for the portfolio is: 26.82.

Diversification

Diversification is the measure of return required to justify any loss of diversification for the specific risk taken by the portfolio. To calculate any loss of diversification it is necessary to first calculate the effective beta required so that the systematic risk is equivalent to the total risk for the portfolio. Referred to as the Fama beta, it is calculated as:

$$\beta_f = \frac{\sigma_p}{\sigma_m}$$

For the portfolio, the Fama beta $\beta_f = 0.69$

Therefore, the return required to justify not being fully diversified is calculated as:

$$\textbf{d} = (\beta_f - \beta_p)\ x\ (r_m - r_f)$$

The return required to justify not being fully diversified for the portfolio is: 0.69 percent or less than one percent, assuming a risk free rate of 5 percent.

V. RELATIVE RISK

The risk measures up to this point are absolute rather than relative risk measures. More specifically, the returns and risk of the portfolio and the S&P 500 index are calculated separately and then used for comparison.

Relative risk measures focus on the excess return of the portfolio against the benchmark or in this case the S&P 500 index. The variability of excess return calculated using standard deviation is called tracking error, tracking risk, relative risk or active risk.

Tracking Error

Tracking error is a measure of how closely a portfolio follows the index to which it is benchmarked. The most common measure is the root-mean-square of the difference between the portfolio and index returns.

The tracking error allows the risk differentials between the portfolio and the benchmark S&P 500 index to be measured. It is defined as the standard deviation of the excess return between the portfolio and the S&P 500. For the purpose of this analysis only the *ex post* tracking error is being considered.

$$\textbf{Tracking error} = \sqrt{\frac{\sum_1^n (a_i - \bar{a})^2}{n}}$$

Where a_i excess return and $\bar{a}$ is the mean excess return. This is also the standard deviation of the series of excess returns.

The tracking error for the portfolio is: 9.93

The lower the value of the tracking error, the closer the risk of the portfolio is to the risk of the benchmark. Benchmarked management requires the tracking error to remain below a certain threshold, which if fixed in advance.

Any additional return obtained, as measured by alpha, must also be sufficient to make up for the additional risk undertake by the portfolio. This is in turn determined by the Information ratio.

Information Ratio

The Information ratio is a measure of the risk-adjusted return of the portfolio. It is defined as the excess return divided by the tracking error where excess return is the difference between the return of the portfolio and the return of S&P 500 index.

The information ratio is similar to the Sharpe ratio. However the Sharpe ratio compares the excess return of an asset against the risk free return, the information ratio compares mean active returns to the most relevant benchmark index on a per unit of active risk basis. That is to say, the Sharpe ratio equals the information ratio where the benchmark is a risk-free asset.

$$\text{Information Ratio} = \frac{\text{Excess Return}}{\text{Tracking Error}}$$

More specifically, the information ratio is mean of the return on the portfolio less the mean return of the S&P 500, divided by the standard deviation of the differences between the portfolio's return series and that of the S&P 500, also known as the tracking error above. Another way of looking at the information ratio is that it is the residual return not explained by benchmark divided by the residual risk or the standard deviation of the excess returns.

The information ratio for the portfolio is: 0.52

Grinold and Kahn state that an information ratio of 0.75 is very good and a 1.0 is exceptional.

VI. RETURN DISTRIBUTION CHARACTERISTICS

Skewness

The skewness of the distribution indicates if the distribution of returns for the portfolio is positively or negatively skewed.

The skewness of the portfolio is: 0.-75

With a skewness of -0.75, we can see that the portfolio's returns are slightly negatively skewed.

The S&P 500 on a monthly basis is also negatively skewed at -0.82.

It is interesting to note that if we remove the negative 2008 return of a -19.15 percent, the skew becomes a positive 0.01.

Kurtosis (Pearson's kurtosis)

Kurtosis provides information regarding the shape of the portfolio's return distribution and is generally associated with how flat or peaked the distribution is. Note that a normal distribution has a kurtosis of 3 and most kurtosis calculations show the excess above or below this number.

The excess kurtosis of the portfolio is: 1.22 and is Leptokurtic - heavy in the tails

VII. RISK-ADJUSTED PERFORMANCE MEASURES

Drawdown

The simplest measure of risk in any return series from an absolute return investor's perspective, assuming he or she wishes to avoid losses, is any continuous losing return period or drawdown. There are many flavors of drawdown. We are defining a drawdown as a negative return by the portfolio. For greater accuracy, drawdown is calculated using the monthly returns of the certified data.

Average Drawdown

The average drawdown is the average of negative returns over the investment period.

Average drawdown $\bar{D}$ is the absolute value of $\sum_{j=1}^{j=d} \frac{D_j}{d}$

Where: D_j = jth drawdown over the period
Where: d = the total number of drawdowns in the entire period.

The average drawdown for the portfolio is: 4 percent.

Maximum Drawdown

The maximum drawdown (D_{max}), not to be confused with the largest individual drawdown, is the maximum potential loss over a specific time period, in this case the period under consideration. Maximum drawdown represents the maximum loss an investor can suffer by buying in at the highest point and selling at the lowest.

Maximum drawdown for the portfolio is: 24.5 percent.

Largest Individual Drawdown

As its name implies, the largest individual drawdown (D_{Lar}) is the largest individual uninterrupted loss in a return series.

Largest individual drawdown for the portfolio it is: 24.5 percent.

Recovery Time (or drawdown duration)

The recovery time or drawdown duration is the time taken to recover from an individual or maximum drawdown to the original level.

As of December 31, 2008, the portfolio had not achieved its previous high point.

Drawdown Deviation

Drawdown deviation calculates a standard deviation-type statistic using individual drawdowns. It is calculated as:

$$\text{Drawdown deviation DD} = \sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}}$$

Where D_j = j^{th} drawdown over the entire series

The drawdown deviation for the portfolio is: 2.853

VIII. DOWNSIDE RISK OR SEMI-STANDARD DEVIATION

Downside Risk

Standard deviation and the symmetrical normal distribution are the foundations of modern portfolio theory. Post-modern portfolio theory recognizes that investors prefer upside risk rather than downside risk and therefore utilizes the semi-standard deviation.

Semi-standard deviation measures the variability of underperformance below a minimum target rate. The minimum target rate for our purposes here is 5 percent. In other words, the downside risk only takes into account negative returns. All positive returns are included as zero in the calculation of semi-standard deviation or downside risk as follows:

$$\text{Downside risk } \sigma_D = \sqrt{\sum_{i=1}^{n} \frac{\min[(r_i - r_t), 0]^2}{n}}$$

The downside risk for the portfolio, also known as the standard deviation of negative returns, is 3.62.

Sortino Ratio

The Sortino ratio is a financial ratio, similar to the Sharpe ratio that measures the risk-adjusted return of investments or portfolios. The numerator is the mean return of the portfolio less a minimum targeted return. The minimum return used here is the risk free rate of 5 percent.

Unlike the Sharpe ratio, the Sortino uses downside-volatility (sometimes referred to as semi-volatility) as the denominator instead of standard deviation. The use of downside-volatility allows the Sortino ratio to measure the return of “negative” volatility.

$$\text{The Sortino ratio} = \frac{r_p - r_T}{\sigma_D}$$

The Sortino ratio for the portfolio is: 3.25

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Schedule A

The rates of return have been prepared in compliance with the following policies and procedures:

1) Returns represent those of a portfolio and do not include actual assets.

2) Portfolio returns are based on published index returns.

3) Individual index component returns are weighted using beginning of period weights and then summed to produce monthly portfolio returns.

4) Monthly portfolio returns are geometrically linked to produce period returns.

Schedule B
Portfolio Performance

For the Period January 31, 1981 through December 31, 2008

1981*	11.24%
1982	36.30%
1983	32.86%
1984	11.10%
1985	25.89%
1986	30.45%
1987	19.98%
1988	16.19%
1989	23.35%
1990	3.08%
1991	31.44%
1992	12.28%
1993	21.28%
1994	0.99%
1995	26.02%
1996	13.58%
1997	26.10%
1998	19.87%
1999	26.02%
2000	13.08%
2001	8.91%
2002	-0.61%
2003	33.25%
2004	13.46%
2005	6.68%
2006	14.94%
2007	10.93%
2008	-19.15%

* Not a full annual period.

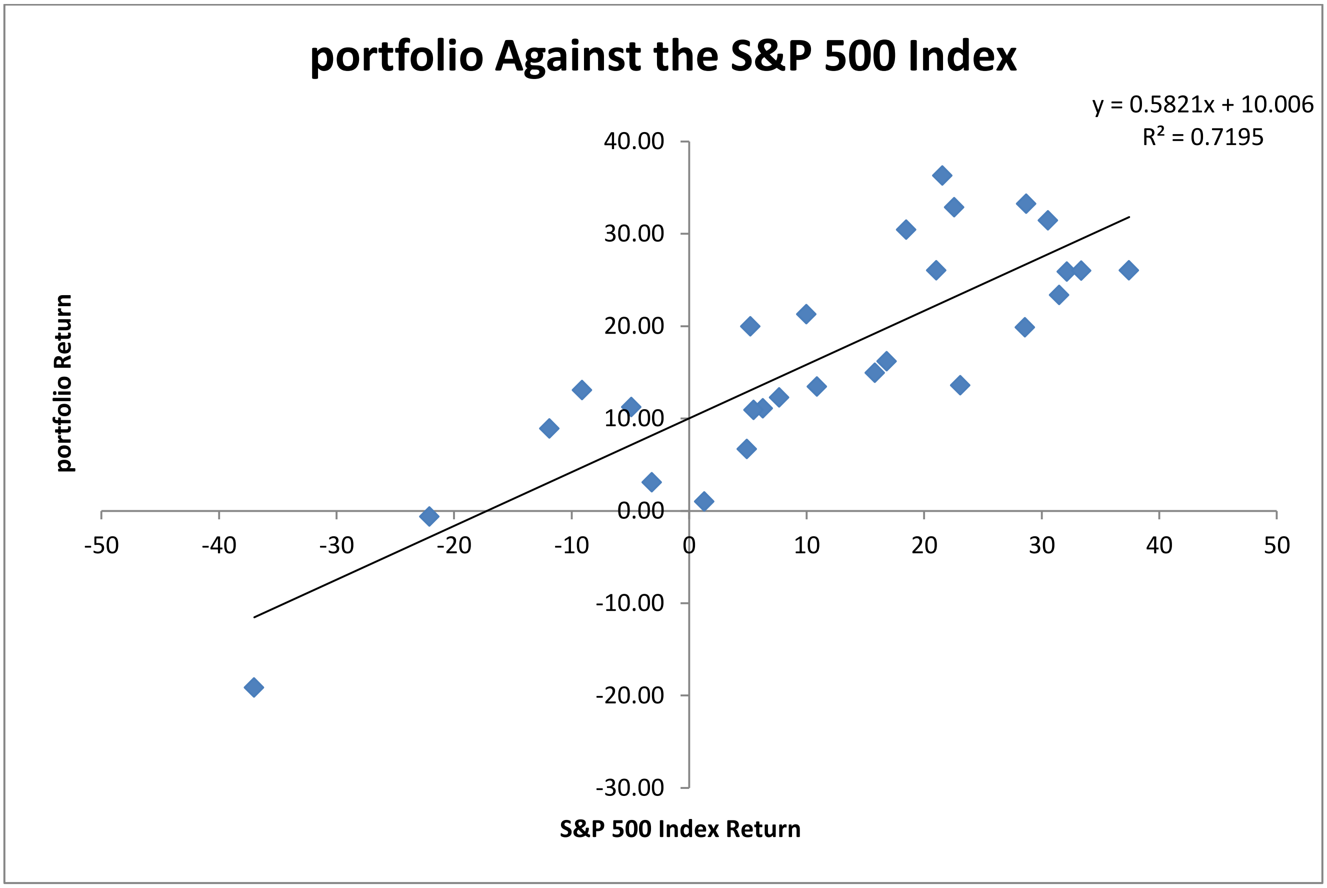
Appendix A – Summary Table

Up Capture	1.11 or 111percent
Down Capture	-0.136 or -13.6 percent
Up Number	100 percent
Down Number	33.3 percent
Up Percentage Ratio	59 percent
Down Percentage Ratio	100 percent
Percentage Gain Ratio	118 percent
portfolio Variance	151
S&P 500 Index Variance	321
portfolio Standard deviation	12.3
S&P 500 Standard deviation	17.9
portfolio Sharpe Ratio (arithmetic)	0.96
S&P 500 Sharpe Ratio (arithmetic)	0.65
portfolio Sharpe Ratio (geometric)	0.90
S&P 500 Sharpe Ratio (geometric)	0.56
M^2	22.14 percent
Excess Return M^2	10.3 percent
Regression Equation	Y = 10.006 + 0.5821 X
Beta	0.5821

Jensen's Alpha	15.61
Bull Beta	0.607
Bear Beta	0.823
Beta Timing Ratio	0.74
portfolio Mean Return	16.77
S&P 500 Mean Return monthly	11.61
Correlation Ratio (portfolio & S&P 500)	0.85
R^2	0.72
Systematic Risk	10.43
Specific or Residual Risk Std. Dev (σ_e)	6.51
Treynor Ratio	8.18
Modified Treynor	1.13
Appraisal ratio (Treynor-Black)	2.4
Modified Jensen	26.82
Non Diversification Penalty	0.69 percent assuming a risk free rate of 5%
Tracking Error or Std. Dev. of excess returns	9.93
Information Ratio	0.52
Skew of portfolio	-0.75
Skew of portfolio without 2008 data	0.01
Skew of S&P 500	-0.82
Excess Kurtosis portfolio	1.22
Average Drawdown	4 percent

Maximum Drawdown	24.5 percent
Largest Individual Drawdown	25 percent
Recovery Time	NA
Drawdown Deviation	2.85
portfolio Downside Risk	3.62
Sortino Ratio	3.25

Appendix B – Regression Graph



Appendix C - Spreadsheet Data